THE NEW JERSEY
ITALIAN HERITAGE COMMISSION

Fibonacci and His Impact on Nature

Grade Level: 6-8

Time Required: Multiple forty minute periods or assign part as homework

Materials Needed:
Internet access, graph paper, poster board, optional - interactive white board and items from nature (spiral sea shells, daisies, sunflowers, pine cones, beehive, various leaves); ruler, compass

Objectives:
Students will be able to:

1. identify Fibonacci as one of the central influences to the widespread use of the Hindo-Arabic numerical system and of mathematical formulas used in western education and commercial applications.
2. briefly describe Fibonacci's life and accomplishments.
3. develop connections between Fibonacci's Sequence, The Golden Rectangle, The Golden Spiral and various items in nature.
4. infer what modern mathematics may be like had Fibonacci not traveled to North Africa.

Standards: https://www.storyofmathematics.com/medieval_fibonacci.html
Please read the New Jersey Student Learning Standards on page 7 before conducting the lesson. They will help you give explicit instructions to your students and help you create rubrics most appropriate for your class.

Procedures:

1. Optional Previous Night's Homework: The students should briefly research the Middle Ages (6^{th} to the 16^{th} Centuries / about 500 A.D. to about 1500 A.D.) with a focus on the history of the Roman numerical system. Roman Numerals were the primary numerical system of Europe during that time.
2. As a follow up to the Previous Night's Homework, students should discuss their findings in small groups and read the Historical Background component for Leonardo Pisano Bigollo.
3. From the Historical Background, the students should be able to map the progression of numerical calculations of the early 1200s as introduced to Europe by Leonardo Pisano Bigollo also known in later years as "Leonardo Fibonacci."
4. The students should solve Fibonacci's Sequence of numbers by illustrating the number pattern from the rabbit "word problem" referenced in the Historical Background.

"A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How

many pairs of rabbits will be produced in a year, beginning with a single pair, if in every month each pair bears a new pair which becomes productive from the second month on?"

5. The answer to this problem creates a sequence of numbers. Students should identify and understand the calculation to achieve the sequence. This pattern is later called the Fibonacci Sequence.
    a. 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144…
6. The students should draw connections (through research) of how Fibonacci's number sequence is also prevalent in geometric terms.
    a. The students should experiment with Fibonacci numbers and how they connect to squares of a geometric pattern. This is called the **Golden Rectangle**.
        i. The ratio for a Golden Rectangle is $\frac{1+\sqrt{5}}{2}$, or approximately 1.618. Students can create true Golden Rectangles using graph paper, a ruler, and a compass. They can then divide this first rectangle, creating new ones.
        ii. Students and/or the teacher can consult the following website for assistance: http://jwilson.coe.uga.edu/emt669/student.folders/may.leanne/leanne's%20page/golden.ratio/golden.ratio.html
        iii. Students can also approximate the Golden Rectangle using graph paper. Using a poster board, graph paper, white board, or interactive white board, the teacher should illustrate how the Fibonacci numbers indicate how to multiply a simple 1x1 square.
            1. 1x1, 2x1, 3x2, 5x3, 8x5, 13x8, 21x13, 34x21…
            2. As they construct rectangles, have students calculate the ratios of the sides:
                a. 1x1=1; 2x1=2; 3x2=1.5; 5x3=1.667; etc.
                b. Notice how, as the rectangles get larger, the ratio gets closer to the Golden Ratio.
                    i. Students can calculate the ratios of large rectangles to prove that the Golden Ratio is the limit of the Fibonacci rectangles.
                c. For a visual of this, see: http://math.rice.edu/~lanius/Geom/building.html
        iv. The students and/or teacher may wish to refer to the following website or others similar to provide a visual of the Golden Rectangle. http://jwilson.coe.uga.edu/EMT668/EMAT6680.2000/Obara/Emat6690/Golden%20Ratio/golden.html

            http://www.mathopenref.com/rectanglegolden.html

            http://www.miniwebtool.com/golden-rectangle-calculator/
    b. Once the above task has been completed, the students may explore how to create the **Golden Spiral** within the Golden Rectangle.
        i. The Golden Spiral can be created by first using the Fibonacci sequence to create the Golden Rectangle.

ii. The spiral can be drawn by beginning either in the smallest square of the rectangle. Begin your drawing point in the top corner of the square. Draw a smooth continuous arc through each addition square connecting its opposing corners until you reach the largest and outer most square of the Golden Rectangle.

iii. Explain to the students that this spiral can be seen in many items within nature, for example a nautilus (sea shell). The students may wish to view a picture of this type of sea shell via internet and highlight the spiral using possibly a computer or interactive white board. If available, students may examine real sea shells.

iv. The students may then try to create the Golden Spiral on their own using paper/pencil or computer soft ware.

v. For further information and a visual direction on the Golden Spiral, refer to the following website or others that are similar using internet resources.

http://jwilson.coe.uga.edu/EMT668/EMAT6680.2000/Obara/Emat6690/Golden%20Ratio/golden.html

http://www.goldennumber.net/spirals/

https://www.wikihow.com/Draw-the-Golden-Spiral

7. The students should begin to make the connection of these patterns to various aspects of nature as given below. Identify the Fibonacci sequence, the Golden Rectangle or Golden Spiral.
   a. The teacher may illustrate these patterns together as a class via interactive white board or by putting students into small groups to illustrate one pattern. The teacher may have the students use poster board or graph paper to illustrate.
   b. Students may complete the following assignment either with or without any help from internet research as the teacher determines. Examining sunflowers, daisies, bee hive, pine cones or various leaves would connect to various learning modalities and aid in student comprehension.
      i. Honeybee Hives
      ii. Student Family Trees
         1. How many children, over the course of a few generations, could an initial set of parents produce?
      iii. Petals on a Flower
         1. Most species of daisies have a total of 34, 55 or 89 petals. (Fibonacci numbers)
         2. Using the Fibonacci Sequence beginning with 3, identify various types of flowers that have the same number of petals as a Fibonacci number.
      iv. Pine cones
      v. Leaf arrangements

1. Is it more common to find a three or four leaf clover? Why? Are you familiar with the legend of the four leaf clover? How does the legend connect to Fibonacci?

c. Suggested internet resources:
    i. https://www.livescience.com/37704-phi-golden-ratio.html

       https://www.britannica.com/science/golden-ratio

       https://www.omnicalculator.com/math/golden-ratio

    ii. The Fibonacci Series –Applications – Nature
       https://www.simpleway2code.me/2018/11/series-of-fibonacci-and-prime-numbers.html

       https://www.tutorialgateway.org/python-fibonacci-series-program/

d. As a class, share and discuss your findings.

Homework/Assessment:
Students should create a brief writing response to one of the following:
1) Describe Fibonacci accomplishments and how he impacted modern society. What do you think mathematics would be like today for students had Fibonacci not traveled to North Africa and shared his findings?
2) Describe how nature follows various patterns or sequences. Why do you think this is so?

Historical Background

The twelfth century, also known as the 1100s or the High Middle Ages, was a lively period in European history, notably for the rise of universities, and the contributions of Italian maritime republics like Genoa, Pisa, and Amalfi.

Leonardo Pisano Bigollo, "Fibonacci," has been referred to as the most advanced and talented mathematician of the Middle Ages. He is credited for inspiring the wide-spread use of the Hindo-Arabic number system throughout Europe. He was believed to have been born sometime between 1170 A.D. and 1175 A.D. in the Republic of Pisa, now a part of modern day Italy. Some historians believe the last name "Bigollo" was used as a nickname and may mean "traveler" or "good-for-nothing." It has been suggested by historians that perhaps some people of Bigollo's day believed this meaning suited him well. He was indeed a world traveler with his father, (Guglielmo Bonacci, an important trade ambassador), but also immersed himself in the study of arithmetic, which many considered to have no practical value. The name "Fibonacci" was not given to him until centuries after his death by Guillaume Libri. The nickname was formed from his father's name and the Latin root "filius Bonacci" meaning "son of Bonacci." Fibonacci is also known as Leonardo of Pisa, Leonardo Pisano, and Leonardo Bonacci.

As a young boy, Fibonacci traveled to join his father in Bugia, North Africa, which is present day Bejaia, Algeria. His father had been appointed to serve as a public notary on behalf of the Republic of Pisa. It was there that Fibonacci was educated by the Moors, who were an Arabic society of nomadic people. As a citizen of Latin speaking society, he was familiar with using the Roman numerical system which used letters that correlated to specific values. Through his education and many encounters with merchants in Bugia, Fibonacci was introduced to the "Hindo-Arabic" system of numerals. This system is based on nine digits, 1 – 9, zero and a decimal system whereby introducing a place value system. Fibonacci soon realized the value of this new system. It had fewer numerical symbols and made the calculation process more efficient and accurate than using the Roman numeral system.

Upon his return to Pisa, Fibonacci shared the concept of this new Arabic or "Indian" numerical system with society by authoring *Liber abbaci* in 1202 as Leonardo Pisano. *Liber abbaci* can be translated as "Book of Calculations" or "abacus." In this text, he provided a detailed explanation of algebraic theory. Merchants and other members of society, who frequently used accounting, also recognized the advantages of the Hindo-Arabic numerical system. In his book, Fibonacci posed mathematical questions, what students today might consider word problems. One of which inquires, *"How many pairs of rabbits will be produced in a year, beginning with a single pair, if in every month each pair bears a new pair which becomes productive from the second month on?"* The answer to this problem creates a sequence of numbers which is later called the Fibonacci Series. The sequence is unending. The key to this sequence is adding the two previous numbers to achieve the next. In his book, Fibonacci explains in depth the answer to this question.

In summary, beginning with just one pair in the first month and using the formula stated above, by the end of the twelfth month, there would be 377 pairs of rabbits. It has not been confirmed whether Fibonacci invented these word problems or if they were questions he encountered during his studies abroad and included as part of his book. This first book also introduced lattice multiplication, place value, and Egyptian fractions. Many of these concepts are now a part of modern day mathematical lessons for young students. This book was later revised by Fibonacci following his meeting with the Holy Roman Emperor. Fibonacci developed his second text in 1220 titled *Practica geometriae*. The title of this book may not be difficult to translate, *The Practice of Geometry*. This book provided an understanding of practical theorems and proofs for surveyors.

Fibonacci's work drew the attention of scholars of the Holy Roman Emperor Frederick II. They in turn convinced Frederick II that he should meet with Fibonacci. During this meeting, Frederick's scholars developed many mathematical challenges for Fibonacci to solve. This prompted Fibonacci to write his third book, *Flos,* in 1225 which means the "Flower of Mathematics." It was within this book that Fibonacci provided detailed explanation of the mathematical questions he was challenged to solve. Fredrick II and his scholars were impressed with Fibonacci's solutions and his knowledge of mathematics. They praised him as "*the serious and learned Master Leonardo Bigollo…"* for sharing his knowledge with citizens and providing guidance to the Republic of Pisa on matters of accounting.

Fibonacci's final book in 1225, *Liber quadratorum*, was considered his most impressive, yet not most famous, endeavor. This book was the Book of Squares and explores Pythagorean triples. Fibonacci did write other texts, *Di minor guise and Elements,* but because of the time era, additional copies were handwritten and very few in number. Many copies have been lost over the course of time.

Fibonacci is believed to have died approximately 1240 A.D. His contributions to the world of mathematics has spanned for almost 800 years. His journals and mathematical texts provide insight to help modern societies understand more about life during the Middle Ages, important merchandise, ports for import and export, monetary conversions, cities that minted money, etc. Streets in both Pisa and Florence have been named in his honor. A statue of Fibonacci is located near the Cathedral in Pisa and also in Florence.

Background Resources:

Biography of Leonardo Pisano Fibonacci, Noted Italian Mathematician, Think Co.
https://www.thoughtco.com/leonardo-pisano-fibonacci-biography-2312397
Hordadam, A.F. "Eight Hundred Years Young." The Australian Mathematics Teacher 31 (1975).
Kimberling, Clark, Evansville University, Indiana.
<http://faculty.evansville.edu/ck6/bstud/fibo.html >
MEDIEVAL MATHEMATICS – FIBONACCI,
https://www.storyofmathematics.com/medieval_fibonacci.html
O'Connor, JJ; Robertson, EF. "Leonardo Pisano Fibonacci." MacTutor History of Mathematics. October 1998. http://www-history.mcs.st-andrews.ac.uk/Biographies/Fibonacci.html
Knotts, Dr. Ron. "Who was Fibonacci?" September 28, 2009.
http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibBio.html

New Jersey Student Learning Standards

Social Studies

6.2.8.C.4.e Determine the extent to which interaction between the Islamic world and medieval Europe increased trade, enhanced technology innovation, and impacted scientific thought and the arts.
6.2.8.D.4.j Compare the major technological innovations and cultural contributions of the civilizations of this period and justify which represent enduring legacies.

English Language Arts

RH.6-8.7 Integrate visual information (e.g., in charts, graphs, photographs, videos or maps) with other information in print or digital text.
WHST.6-8.1d Provide a concluding statement or section that follows from and supports the argument.
WHST.6-8.2 Write informative / explanatory texts, including the narration of historical events / scientific procedures / experiments / technical processes.
WHST.6-8.7 Conduct short research projects to answer a question (including self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration.

Mathematics

Interpreting Functions F-IF A. Understand the concept of a function and use function notation 1.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$